EXERCISES

2.1-1. Let the p.m.f. of X be defined by $f(x) = x/9$, $x = 2, 3, 4$.

(a) Draw a bar graph for this p.m.f.

(b) Draw a probability histogram for this p.m.f.

2.1-2. Let a chip be taken at random from a bowl that contains six white chips, three red chips, and one blue chip. Let the random variable $X = 1$ if the outcome is a white chip, let $X = 5$ if the outcome is a red chip, and let $X = 10$ if the outcome is a blue chip.

(a) Find the p.m.f. of X.

(b) Graph the p.m.f. as a bar graph.

2.1-3. For each of the following, determine the constant c so that $f(x)$ satisfies the conditions of being a p.m.f. for a random variable X, and then depict each p.m.f. as a bar graph:

(a) $f(x) = x/c, \quad x = 1, 2, 3, 4.$

(b) $f(x) = cx, \quad x = 1, 2, 3, \ldots, 10.$

(c) $f(x) = c(1/4)^x, \quad x = 1, 2, 3, \ldots.$

(d) $f(x) = c(x+1)^2, \quad x = 0, 1, 2, 3.$

(e) $f(x) = x/c, \quad x = 1, 2, 3, \ldots, n.$

(f) $f(x) = \dfrac{c}{(x+1)(x+2)}, \quad x = 0, 1, 2, 3, \ldots.$

x-1 would be a better problem to get geometric rv.

2.1-4. The state of Michigan generates a three-digit number at random six days a week for its daily lottery. The numbers are generated one digit at a time. Consider the following set of 50 three-digit numbers as 150 one-digit integers that were generated at random:

169	938	506	757	594	656	444	809	321	545
732	146	713	448	861	612	881	782	209	752
571	701	852	924	766	633	696	023	601	789
137	098	534	826	642	750	827	689	979	000
933	451	945	464	876	866	236	617	418	988

Let X denote the outcome when a single digit is generated.

(a) With true random numbers, what is the p.m.f. of X? Draw the probability histogram.

(b) For the 150 observations, determine the relative frequencies of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, respectively.

(c) Draw the relative frequency histogram of the observations on the same graph paper as that of the probability histogram. Use a colored or dashed line for the relative frequency histogram.

2.1-5. The p.m.f. of X is $f(x) = (5 - x)/10$, $x = 1, 2, 3, 4$.

(a) Graph the p.m.f. as a bar graph.

(b) Use the following independent observations of X, simulated on a computer, to construct a table like Table 2.1-1:

3	1	2	2	3	2	2	2	1	3	3	2	3	2	4	4	2	1	1	3
3	1	2	2	1	1	4	2	3	1	1	1	2	1	3	1	1	3	3	1
1	1	1	1	1	4	1	3	1	2	4	1	1	2	3	4	3	1	4	2
2	1	3	2	1	4	1	1	1	2	1	3	4	3	2	1	4	4	1	3
2	2	2	1	2	3	1	1	4	2	1	4	2	1	2	3	1	4	2	3

(c) Construct a probability histogram and a relative frequency histogram like Figure 2.1-3.

2.1-6. Let a random experiment be the casting of a pair of unbiased dice, each having six faces, and let the random variable X denote the sum of the dice.

(a) With reasonable assumptions, determine the p.m.f. $f(x)$ of X. HINT: Picture the sample space consisting of the 36 points (result on first die, result on second die), and assume that each has probability 1/36. Find the probability of each possible outcome of X, namely, $x = 2, 3, 4, \ldots, 12$.

(b) Draw a bar graph for $f(x)$.

2.1-7. Let a random experiment be the casting of a pair of unbiased six-sided dice, and let X equal the smaller of the outcomes if they are different and the common value if they are equal.

(a) With reasonable assumptions, find the p.m.f. of X.

(b) Draw a probability histogram of the p.m.f. of X.

(c) Let Y equal the range of the two outcomes (i.e., the absolute value of the difference of the largest and the smallest outcomes). Determine the p.m.f. $g(y)$ of Y for $y = 0, 1, 2, 3, 4, 5$.

(d) Draw a probability histogram for $g(y)$.

2.1-8. An unbiased four-sided die X has two faces numbered 0 and two faces numbered 2. An unbiased four-sided die Y has its faces numbered 0, 1, 4, and 5. The two dice are rolled. Let $W = X + Y$.

(a) Determine the p.m.f. of W.

(b) Draw a probability histogram of the p.m.f. of W.

2.1-9. Let the p.m.f. of X be defined by $f(x) = (1 + |x - 3|)/11$, $x = 1, 2, 3, 4, 5$. Graph the p.m.f. of X as a bar graph.

2.1-10. Suppose there are 3 defective items in a lot (collection) of 50 items. A sample of size 10 is taken at random and without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains

(a) Exactly one defective item.

(b) At most one defective item.

2.1-11. In a lot (collection) of 100 light bulbs, there are 5 bad bulbs. An inspector inspects 10 bulbs selected at random. Find the probability of finding at least one defective bulb. HINT: First compute the probability of finding no defectives in the sample.

2.1-12. A lot (collection) of $N_1 + N_2 = 25$ items is accepted if the number of defectives, X, among $n = 5$ items taken at random and without replacement from the lot is less than or equal to 1. The operating characteristic curve is defined as the probability of accepting the lot, namely,

$$\text{OC}(p) = P(X \le 1) = P(X = 0) + P(X = 1),$$

where $p = N_1/25$. Determine the probabilities OC(0.04), OC(0.08), OC(0.12), and OC(0.16) (and compare them with those in Example 2.1-7).

2.1-13. Let X be the number of accidents per week in a factory. Let the p.m.f. of X be

$$f(x) = \frac{1}{(x+1)(x+2)}, \qquad x = 0, 1, 2, \cdots.$$

Find the conditional probability of $X \ge 4$, given that $X \ge 1$.
HINT: Write $f(x) = 1/(x + 1) - 1/(x + 2)$.

2.1-14. Often in buying a product at a supermarket, there is a concern about the item being underweight. Suppose there are 20 "one-pound" packages of frozen ground turkey on display and 3 of them are underweight. A consumer group buys 5 of the 20 packages at random. What is the probability of at least 1 of the 5 being underweight?

2.1-15. A professor gave her students six essay questions, from which she will select three for a test. A student has time to study for only three of these questions. What is the probability that, of the questions studied,

(a) at least one is selected for the test?

(b) all three are selected?

(c) exactly two are selected?

2.1-16. An urn contains 20 chips, 10 marked H and 10 marked C. Randomly select chips, one at a time, without replacement. After 10 chips have been selected, let X equal the difference between the numbers of H and C chips that have been selected.

(a) Define the p.m.f. of X.

(b) What value of X is most likely? That is, what is the mode of X?

REMARK Hope College (H chips) and Calvin College (C chips) began playing basketball in 1920–21. After 166 games, each team had won 83. The 20 chips correspond to the first 20 games only, and assume that Hope and Calvin were tied after 20 games in order to keep the numbers reasonable. ■

2.1-17. Five cards are selected at random without replacement from a standard, thoroughly shuffled 52-card deck of playing cards. Let X equal the number of face cards (kings, queens, jacks) in the hand. Forty observations of X yielded the following data:

2	1	2	1	0	0	1	0	1	1	0	2	0	2	3	0	1	1	0	3
1	2	0	2	0	2	0	1	0	1	1	2	1	0	1	1	2	1	1	0

(a) Argue that the p.m.f. of X is

$$f(x) = \frac{\binom{12}{x}\binom{40}{5-x}}{\binom{52}{5}}, \qquad x = 0, 1, 2, 3, 4, 5$$

and thus, that $f(0) = 2109/8330$, $f(1) = 703/1666$, $f(2) = 209/833$, $f(3) = 55/833$, $f(4) = 165/21{,}658$, and $f(5) = 33/108{,}290$.

(b) Draw a probability histogram for this distribution.

(c) Determine the relative frequencies of $0, 1, 2, 3$, and superimpose the relative frequency histogram on your probability histogram.

2.1-18. (Michigan Mathematics Prize Competition, 1992, Part II) From the set $\{1, 2, 3, \ldots, n\}$, k distinct integers are selected at random and arranged in numerical order (from lowest to highest). Let $P(i, r, k, n)$ denote the probability that integer i is in position r. For example, observe that $P(1, 2, k, n) = 0$, as it is impossible for the number 1 to be in the second position after ordering.

(a) Compute $P(2, 1, 6, 10)$.

(b) Find a general formula for $P(i, r, k, n)$.

2.1-19. A bag contains 144 ping-pong balls. More than half of the balls are painted orange and the rest are painted blue. Two balls are simultaneously drawn at random. The probability of drawing two balls of the same color is the same as the probability of drawing two balls of different colors. How many orange balls are in the bag?

EXERCISES

2.2-1. Find $E(X)$ for each of the distributions given in Exercise 2.1-3.

HINT: Part (c). Note that the difference $E(X) - (1/4)E(X)$ is equal to the sum of a geometric series.

2.2-2. Let the random variable X have the p.m.f.

$$f(x) = \frac{(|x| + 1)^2}{9}, \qquad x = -1, 0, 1.$$

Compute $E(X)$, $E(X^2)$, and $E(3X^2 - 2X + 4)$.

2.2-3. In a particular lottery, 3 million tickets are sold each week for 50¢ apiece. Out of the 3 million tickets, 12,006 are drawn at random and without replacement and awarded prizes: twelve thousand \$25 prizes, four \$10,000 prizes, one \$50,000 prize, and one \$200,000 prize. If you purchased a single ticket each week, what is the expected value of this game to you?

2.2-4. In a state lottery, a three-digit integer is selected at random. A player bets \$1 on a particular number, and if that number is selected, the payoff is \$500 minus the \$1 paid for the ticket. Let X equal the payoff to the bettor, namely, $-\$1$ or \$499, and find $E(X)$.

2.2-5. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f.

$$f(x) = \frac{5 - x}{10}, \qquad x = 1, 2, 3, 4.$$

If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

2.2-6. An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having p.m.f.

$$f(x) = \begin{cases} 0.9, & x = 0, \\ \dfrac{c}{x}, & x = 1, 2, 3, 4, 5, 6, \end{cases}$$

where c is a constant. Determine c and the expected value of the amount the insurance company must pay.

2.2-7. In the gambling game chuck-a-luck, for a \$1 bet it is possible to win \$1, \$2, or \$3 with respective probabilities 75/216, 15/216, and 1/216. One dollar is lost with probability 125/216. Let X equal the payoff for this game and find $E(X)$. Note that when a bet is won, the \$1 that was bet, in addition to the \$1, \$2, or \$3 that is won, is returned to the bettor.

2.2-8. Let the p.m.f. of X be defined by $f(x) = 6/(\pi^2 x^2)$, $x = 1, 2, 3, \ldots$. Show that $E(X)$ does not exist in this case. (Can you show that $f(x)$ is a p.m.f.?)

2.2-9. Let us select at random a number from the first n positive integers. If the payment is equal to the reciprocal of the number, find an expression for the expected payment. Evaluate this number when $n = 5$. Approximate this number when $n = 100$, or find the exact value using the computer. HINT: For the approximation, use a modification of the integral test for testing the convergence of a series.

2.2-10. Let X be a random variable with support $\{1, 2, 3, 5, 15, 25, 50\}$, each point of which has the same probability 1/7. Argue that $c = 5$ is the value that minimizes $h(c) = E(|X - c|)$. Compare c with the value of b that minimizes $g(b) = E[(X - b)^2]$.

2.2-11. A roulette wheel used in a U. S. casino has 38 slots, of which 18 are red, 18 are black, and 2 are green. A roulette wheel used in a French casino has 37 slots, of which 18 are red, 18 are black, and 1 is green. A ball is rolled around the wheel and ends up in one of the slots with equal probability. Suppose that a player bets on red. If a \$1 bet is placed, the player wins \$1 if the ball ends up in a red slot. (The player's \$1 bet is returned.) If the ball ends up in a black or green slot, the player loses \$1. Find the expected value of this game to the player in

(a) The United States.

(b) France.

2.2-12. In the casino game called **high–low**, there are three possible bets. Assume that \$1 is the size of the bet. A pair of fair six-sided dice is rolled and their sum is calculated. If you bet **low**, you win \$1 if the sum of the dice is $\{2, 3, 4, 5, 6\}$. If you bet

high, you win \$1 if the sum of the dice is $\{8, 9, 10, 11, 12\}$. If you bet on $\{7\}$, you win \$4 if a sum of 7 is rolled. Otherwise you lose on each of the three bets. In all three cases, your original dollar is returned if you win. Find the expected value of the game to the bettor for each of these three bets.

2.2-13. In the gambling game craps (see Exercise 1.4-13), the player wins \$1 with probability 0.49293 and loses \$1 with probability 0.50707 for each \$1 bet. What is the expected value of the game to the player?

2.2-14. Suppose that a school has 20 classes: 16 with 25 students in each, three with 100 students in each, and one with 300 students, for a total of 1000 students.

(a) What is the average class size?

(b) Select a student randomly out of the 1000 students. Let the random variable X equal the size of the class to which this student belongs, and define the p.m.f. of X.

(c) Find $E(X)$, the expected value of X. Does this answer surprise you?

EXERCISES

2.3-1. Find the mean and variance for the following discrete distributions:

(a) $f(x) = \dfrac{1}{5}, \qquad x = 5, 10, 15, 20, 25.$

(b) $f(x) = 1, \qquad x = 5.$

(c) $f(x) = \dfrac{4-x}{6}, \qquad x = 1, 2, 3.$

2.3-2. For each of the following distributions, find $\mu = E(X), E[X(X-1)]$, and $\sigma^2 = E[X(X-1)] + E(X) - \mu^2$:

(a) $f(x) = \dfrac{3!}{x!(3-x)!}\left(\dfrac{1}{4}\right)^x\left(\dfrac{3}{4}\right)^{3-x},$
$x = 0, 1, 2, 3.$

(b) $f(x) = \dfrac{4!}{x!(4-x)!}\left(\dfrac{1}{2}\right)^4,$
$x = 0, 1, 2, 3, 4.$

2.3-3. Given $E(X+4) = 10$ and $E[(X+4)^2] = 116$, determine (a) $\text{Var}(X+4)$, (b) μ, and (c) σ^2.

2.3-4. Let μ and σ^2 denote the mean and variance of the random variable X. Determine $E[(X-\mu)/\sigma]$ and $E\{[(X-\mu)/\sigma]^2\}$.

2.3-5. Consider an experiment that consists of selecting a card at random from an ordinary deck of cards. Let the random variable X equal the value of the selected card, where Ace = 1, Jack = 11, Queen = 12, and King = 13. Thus, the space of X is $S = \{1, 2, 3, \cdots, 13\}$. If the experiment is performed in an unbiased manner, assign probabilities to these 13 outcomes and compute the mean μ of this probability distribution.

2.3-6. Place eight chips in a bowl: Three have the number 1 on them, two have the number 2, and three have the number 3. Say each chip has a probability of 1/8 of being drawn at random. Let the random variable X equal the number on the chip that is selected, so that the space of X is $S = \{1, 2, 3\}$. Make reasonable probability assignments to each of these three outcomes, and compute the mean μ and the variance σ^2 of this probability distribution.

2.3-7. A fair coin is flipped successively at random until the first head is observed. Let the random variable X denote the number of flips of the coin that are required. Then the space of X is $S = \{x : x = 1, 2, 3, 4, \ldots\}$. Later we learn that, under certain conditions, we can assign probabilities to these outcomes in S with the function $f(x) = (1/2)^x, x = 1, 2, 3, 4, \cdots$. Compute the mean μ. HINT: Write out the series for μ, and then construct the series for $(1/2)\mu$ and take the difference. An alternative method would be to compare the series for μ with that of the negative binomial $(1-z)^{-2}$, with $z = 1/2$.

2.3-8. Let X equal the number of calls per hour received by 911 between midnight and noon and reported in the *Holland Sentinel*. On October 29 and October 30, the following numbers of calls were reported:

Oct. 29:	0	1	1	1	0	1	2	1	4	1	2	3
Oct. 30:	0	3	0	1	0	1	1	2	3	0	2	2

(a) Find the sample mean.

(b) Find the sample variance.

2.3-9. Let X equal an integer selected at random from the first m positive integers, $\{1, 2, \ldots, m\}$.

(a) Give the values of $E(X)$ and $\text{Var}(X)$.

(b) Find the value of m for which $E(X) = \text{Var}(X)$. (See Zerger in the references.)

2.3-10. Students in a statistics class were asked to report the number of pets in their family. The following data were collected:

3	3	2	4	4	4	3	2	3	2	2	3	4	2	2	3
2	3	4	4	2	3	2	3	3	2	4	3	3	3	2	2
2	2	1	4	4	2	3	3	1	2	2	2	2	3	1	

(a) Find the frequencies of 1, 2, 3, 4.

(b) Calculate the sample mean and the sample standard deviation.

(c) Construct a histogram and locate the mean on the histogram.

2.3-11. Calculate the mean $\bar{x}$ and the variance s^2 of the samples given in

(a) Exercise 1.1-3,

(b) Exercise 1.1-4,

(c) Exercise 1.1-5,

(d) Exercise 1.1-7.

2.3-12. Referring to Exercise 1.1-6, compute the sample mean $\bar{x}$. Could a statistician tell that family about how many boxes of cereal they could expect to purchase to obtain all four prizes?

2.3-13. A measure of **skewness** is defined by

$$\frac{E[(X-\mu)^3]}{\{E[(X-\mu)^2]\}^{3/2}} = \frac{E[(X-\mu)^3]}{(\sigma^2)^{3/2}} = \frac{E[(X-\mu)^3]}{\sigma^3}.$$

When a distribution is symmetrical about the mean, the skewness is equal to zero. If the probability histogram has a longer "tail" to the right than to the left, the measure of skewness is positive, and we say that the distribution is skewed positively, or to the right. If the probability histogram has a longer tail to the left than to the right, the measure of skewness is negative, and we say that the distribution is skewed negatively, or to the left. If the p.m.f. of X is given by $f(x)$, (i) depict the p.m.f. as a probability histogram and find the values of (ii) the mean, (iii) the standard deviation, and (iv) the skewness.

(a)
$$f(x) = \begin{cases} \dfrac{2^{6-x}}{64}, & x = 1, 2, 3, 4, 5, 6, \\[2ex] \dfrac{1}{64}, & x = 7. \end{cases}$$

(b)
$$f(x) = \begin{cases} \dfrac{1}{64}, & x = 1, \\[2ex] \dfrac{2^{x-2}}{64}, & x = 2, 3, 4, 5, 6, 7. \end{cases}$$

2.3-14. In LOTTO 49, Michigan's lottery game, a player selects 6 integers out of the first 49 positive integers. The state then randomly selects 6 out of the first 49 positive integers. Cash prizes are given to a player who matches 4, 5, or 6 integers. Let X equal the number of integers selected by a player that match integers selected by the state.

(a) State the p.m.f. of X.

(b) Calculate the mean, variance, and standard deviation of X.

(c) What value of X is most likely to occur?

(d) On February 25, 1995, the jackpot was worth \$45 million. When the prize is this large, many bets are placed. Out of the 25 million bets that were placed, 3 people matched all six numbers, with each winning \$15 million (most of which was paid by losers during preceding games); 390 matched five numbers, to win \$2500 each and; 22,187 matched four numbers, to win \$100 each. Are these numbers of winners consistent with the probability model?

(e) A mathematics professor convinced some colleagues to pool their LOTTO bets, so that they were able to purchase 138 tickets together. They let the state computer randomly select the numbers on which they placed their bets. Among their 138 bets, 65 matched none of the winning LOTTO numbers, $\{3, 20, 33, 34, 43, 46\}$; 55 matched one, 16 matched two, and 2 matched three numbers. How do these results compare with what they could have expected?

2.3-15. Let X equal the larger outcome when a pair of four-sided dice is rolled. The p.m.f. of X is

$$f(x) = \frac{2x-1}{16}, \qquad x = 1, 2, 3, 4.$$

(a) Find the mean, variance, and standard deviation of X.

(b) Calculate the sample mean, sample variance, and sample standard deviation of the following 100 simulated observations of X, and compare these answers with those found in part (a):

4	4	4	4	2	2	2	3	1	4	3	3	2	3	2	4	4	2	3	4
4	3	4	3	4	3	3	2	4	4	3	2	3	3	3	2	4	4	3	4
1	4	3	4	4	4	3	2	4	4	4	3	1	3	2	4	4	4	4	1
3	4	3	2	4	4	3	3	1	3	3	3	3	2	2	2	3	4	3	3
2	4	2	3	3	2	4	4	3	4	4	4	4	3	4	4	4	4	4	4

(c) Draw the graphs of the probability histogram and the relative frequency histogram on the same figure.

(d) Generate your own data, either rolling (four-sided or six-sided) dice or simulating this experiment on a computer.

2.3-16. A Bingo card has 25 squares with numbers on 24 of them, the center being a free square. The integers that are placed on the Bingo card are selected randomly and without replacement from 1 to 75, inclusive. When a game called "cover-up" is played, balls numbered from 1 to 75, inclusive, are selected randomly and without replacement until a player covers each of the numbers on a card. Let X equal the number of balls that must be drawn to cover all the numbers on a single card.

(a) Argue that the p.m.f. of X, for $x = 24, 25, \ldots, 75$, is

$$f(x) = \frac{\binom{24}{23}\binom{51}{x-24}}{\binom{75}{x-1}} \cdot \frac{1}{75-(x-1)} = \frac{\binom{51}{x-24}}{\binom{75}{x}} \cdot \frac{24}{x} = \frac{\binom{x}{24}}{\binom{75}{24}} \cdot \frac{24}{x}.$$

(b) What value of X is most likely to occur? In other words, what is the mode of this distribution?

(c) To show that the mean of X is $(24)(76)/25 = 72.96$, use the combinatorial identity

$$\binom{k+n+1}{k+1} = \sum_{x=k}^{n+k} \binom{x}{k}.$$

(d) Show that $E[X(X+1)] = \dfrac{24 \cdot 77 \cdot 76}{26} = 5401.8462$.

(e) Calculate $\mathrm{Var}(X) = \mathrm{E}[\mathrm{X}(\mathrm{X}+1)] - \mathrm{E}[\mathrm{X}] - [\mathrm{E}(\mathrm{X})]^2 = \dfrac{46{,}512}{8125} = 5.7246$ and $\sigma = 2.39$.

(f) The following 100 observations of X were simulated on the computer:

75	73	74	74	73	75	75	71	71	74	75	74	74	73	75
75	75	71	68	75	74	72	73	75	74	67	73	71	74	74
73	75	71	73	71	68	74	75	66	75	75	74	75	71	75
72	75	74	75	75	72	75	75	74	73	75	62	64	75	72
74	74	75	73	75	75	73	74	75	68	75	69	74	75	61
73	73	73	72	74	68	74	71	73	75	73	74	72	74	73
75	73	71	70	62	74	74	72	75	74					

Compare these data with the probability model. In particular, make the following comparisons: (i) $\overline{x}$ with μ, (ii) s^2 with σ^2, (iii) s with σ, (iv) $(1/100)\sum_{i=1}^{100} x_i(x_i+1)$ with $E[X(X+1)]$.

(g) Compare the probability histogram with the relative frequency histogram.

2.3-17. To find the variance of a hypergeometric random variable in Example 2.3-5, we used the fact that

$$E[X(X-1)] = \frac{N_1(N_1-1)(n)(n-1)}{N(N-1)}.$$

Prove this result by making the change of variables $k = x - 2$ and noting that

$$\binom{N}{n} = \frac{N(N-1)}{n(n-1)}\binom{N-2}{n-2}.$$

2.3-18. A deck of $n = 10$ cards is numbered from 1 to 10. The cards are shuffled and laid down from left to right, face up. Order each of the five successive pairs of cards. Each of these five pairs determines a random interval. Let X equal the number of these five random intervals that intersect each of the other four intervals. It can be shown that

$$P(X \geq k) = \frac{2^k}{\binom{2k+1}{k}}, \qquad k = 0, 1, 2, 3, 4, 5.$$

(a) Find the probability that at least one interval intersects all the other intervals. (You could also simulate this.)

(b) Prove that $\mu = E(X) = \sum_{k=1}^{5} P(X \geq k)$.

(c) Find the value of $\mu = E(X)$ with $n = 10$ cards.

(d) Let the number of cards increase without bound. Find the value of

$$\mu = \sum_{k=0}^{\infty} \frac{2^k}{\binom{2k+1}{k}}.$$

2.3-19. A warranty is written on a product worth \$10,000 so that the buyer is given \$8000 if it fails in the first year, \$6000 if it fails in the second, \$4000 if it fails in the third, \$2000 if it fails in the fourth, and zero after that. The probability of the product's failing in a year is 0.1; failures are independent of those of other years. What is the expected value of the warranty?

EXERCISES

2.4-1. An urn contains 7 red and 11 white balls. Draw one ball at random from the urn. Let $X = 1$ if a red ball is drawn, and let $X = 0$ if a white ball is drawn. Give the p.m.f., mean, and variance of X.

2.4-2. Suppose that in Exercise 2.4-1, $X = 1$ if a red ball is drawn and $X = -1$ if a white ball is drawn. Give the p.m.f., mean, and variance of X.

2.4-3. On a six-question multiple-choice test there are five possible answers for each question, of which one is correct (C) and four are incorrect (I). If a student guesses randomly and independently, find the probability of

(a) Being correct only on questions 1 and 4 (i.e., scoring C, I, I, C, I, I).

(b) Being correct on two questions.

2.4-4. Suppose that the percentage of college students who engaged in binge drinking, which is defined as having five drinks for male students and four drinks for female students at one "drinking occasion" during the previous two weeks, is approximately 40%. Let X equal the number of students in a random sample of size $n = 12$ who binge drink.

(a) Find the probability that X is at most 5.

(b) Find the probability that X is at least 6.

(c) Find the probability that X is equal to 7.

(d) Give the mean, variance, and standard deviation of X.

2.4-5. In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metal–metal bond. The probability of such a bond forming is $p = 0.20$. Let X equal the number of successful reactions out of $n = 25$ such experiments.

(a) Find the probability that X is at most 4.
(b) Find the probability that X is at least 5.
(c) Find the probability that X is equal to 6.
(d) Give the mean, variance, and standard deviation of X.

2.4-6. It is claimed that 15% of the ducks in a particular region have patent schistosome infection. Suppose that seven ducks are selected at random. Let X equal the number of ducks that are infected.

(a) Assuming independence, how is X distributed?
(b) Find (i) $P(X \geq 2)$, (ii) $P(X = 1)$, and (iii) $P(X \leq 3)$.

2.4-7. Suppose that 2000 points are selected independently and at random from the unit square $\{(x,y) : 0 \leq x < 1, 0 \leq y < 1\}$. Let W equal the number of points that fall into $A = \{(x,y) : x^2 + y^2 < 1\}$.

(a) How is W distributed?
(b) Give the mean, variance, and standard deviation of W.
(c) What is the expected value of $W/500$?
(d) Use the computer to select 2000 pairs of random numbers. Determine the value of W and use that value to find an estimate for π. (Of course, we know the real value of π, and more will be said about estimation later on in this text.)
(e) How could you extend part (d) to estimate the volume $V = (4/3)\pi$ of a ball of radius 1 in 3-space?
(f) How could you extend these techniques to estimate the "volume" of a ball of radius 1 in n-space?

2.4-8. It is believed that approximately 65% of Americans under the age of 65 have private health insurance. Suppose this is true, and let X equal the number of Americans under age 65 in a random sample of $n = 15$ with private health insurance.

(a) Find the probability that X is at least 10.
(b) Find the probability that X is at most 10.
(c) Find the probability that X is equal to 10.
(d) How is X distributed?
(e) Give the mean, variance, and standard deviation of X.

2.4-9. Suppose that the percentage of American drivers who are multitaskers (e.g., talk on cell phones, eat a snack, or text message at the same time they are driving) is approximately 80%. In a random sample of $n = 20$ drivers, let X equal the number of multitaskers.

Do by hand and use R.

(a) How is X distributed?
(b) Give the values of the mean, variance, and standard deviation of X.
(c) Find (i) $P(X = 15)$, (ii) $P(X > 15)$, and (iii) $P(X \leq 15)$.

2.4-10. A boiler has four relief valves. The probability that each opens properly is 0.99.

(a) Find the probability that at least one opens properly.
(b) Find the probability that all four open properly.

2.4-11. A random variable X has a binomial distribution with mean 6 and variance 3.6. Find $P(X = 4)$.

2.4-12. A certain type of mint has a label weight of 20.4 grams. Suppose that the probability is 0.90 that a mint weighs more than 20.7 grams. Let X equal the number of mints that weigh more than 20.7 grams in a sample of eight mints selected at random.

(a) How is X distributed if we assume independence?
(b) Find (i) $P(X = 8)$, (ii) $P(X \leq 6)$, and (iii) $P(X \geq 6)$.

2.4-13. It is given that the probability of germination of each beet seed is 0.8. Thus, if X denotes the number of seeds that germinate in a set of nine seeds, then X is $b(9, 0.8)$, provided that the assumption of independence is valid.

(a) Give the values of μ, σ^2, and σ.
(b) Draw the probability histogram for the p.m.f. of X.
(c) If 100 sets of nine beet seeds were planted, calculate $\bar{x}$, s^2, and s for the following 100 observations:

7	9	7	7	9	7	9	8	5	8	8	6	8	7	8	8	9	7	8	8
8	7	8	7	7	7	6	7	9	7	6	7	6	7	6	5	7	8	7	4
8	8	7	6	9	7	8	7	7	6	7	9	6	6	8	8	8	7	9	5
7	4	7	8	8	9	6	7	6	8	7	5	7	7	9	7	7	8	7	9
7	5	9	8	7	7	8	9	8	5	9	9	8	6	9	8	8	8	9	7

(d) Superimpose the relative frequency histogram of these observations on the graph of the probability histogram.

2.4-14. In the casino game chuck-a-luck, three unbiased six-sided dice are rolled. One possible bet is \$1 on fives, and the payoff is equal to \$1 for each five on that roll. In addition, the dollar bet is returned if at least one five is rolled. The dollar that was bet is lost only if no fives are rolled. Let X denote the payoff for this game. Then X can equal $-1, 1, 2$, or 3.

(a) Determine the p.m.f. $f(x)$.

(b) Calculate μ, σ^2, and σ.

(c) Depict the p.m.f. as a probability histogram.

(d) Calculate $\overline{x}$, s^2, and s, using the 100 following simulated observations of X:

−1	1	−1	−1	1	−1	−1	−1	−1	1
−1	1	−1	2	1	−1	−1	−1	1	−1
1	−1	−1	1	1	−1	−1	−1	−1	1
−1	−1	2	1	1	−1	−1	1	−1	1
−1	−1	1	2	−1	−1	1	−1	−1	−1
−1	1	−1	−1	−1	1	−1	1	−1	2
−1	1	−1	1	−1	2	1	−1	−1	1
−1	1	1	−1	1	1	−1	3	1	−1
−1	−1	1	1	1	1	−1	−1	2	−1
1	−1	1	−1	1	2	−1	1	1	−1

(e) Superimpose the relative frequency histogram of these observations on the probability histogram.

2.4-15. It is claimed that for a particular lottery, 1/10 of the 50 million tickets will win a prize. What is the probability of winning at least one prize if you purchase (a) 10 tickets or (b) 15 tickets?

2.4-16. For the lottery described in Exercise 2.4-15, find the smallest number of tickets that must be purchased so that the probability of winning at least one prize is greater than (a) 0.50; (b) 0.95.

2.4-17. Construct a table that gives the probabilities for the $b(8, 0.2)$ distribution, the hypergeometric distribution with $N_1 = 8$, $N_2 = 32$, and $n = 8$, and the hypergeometric distribution with $N_1 = 16$, $N_2 = 64$, and $n = 8$. (See Figure 2.1-2.)

2.4-18. A hospital obtains 40% of its flu vaccine from Company A, 50% from Company B, and 10% from Company C. From past experience, it is known that 3% of the vials from A are ineffective, 2% from B are ineffective, and 5% from C are ineffective. The hospital tests five vials from each shipment. If at least one of the five is ineffective, find the conditional probability of that shipment's having come from C.

2.4-19. Many products can operate if, out of n parts, k of them are working. Say $n = 20$ and $k = 17$, $p = 0.05$ is the probability that a part fails, and assume independence. What is the probability that the product operates?

2.4-20. A company starts a fund of M dollars from which it pays \$1000 to each employee who achieves high performance during the year. The probability of each employee achieving this goal is 0.10 and is independent of the probabilities of the other employees doing so. If there are $n = 10$ employees, how much should M equal so that the fund has a probability of at least 99% of covering those payments?

2.4-21. A businessman has an important meeting to attend, but he is running a little late. He can take one route to work that has six stoplights or another, longer route that has two stoplights. He figures that if he stops at more than half of the lights on either route, he will be late for the meeting. Assume independence, and assume that the chance of stopping at each light is $p = 0.5$. Which route should he take?

2.4-22. In group testing for a certain disease, a blood sample was taken from each of n individuals and part of each sample was placed in a common pool. The latter was then tested. If the result was negative, there was no more testing and all n individuals were declared negative with one test. If, however, the combined result was found positive, all individuals were tested, requiring $n + 1$ tests. If $p = 0.05$ is the probability of a person's having the disease and $n = 5$, compute the expected number of tests needed, assuming independence.

2.4-23. Your stockbroker is free to take your calls about 60% of the time; otherwise he is talking to another client or is out of the office. You call him at five random times during a given month. (Assume independence.)

(a) What is the probability that he will take every one of the five calls?

(b) What is the probability that he will accept exactly three of your five calls?

(c) What is the probability that he will accept at least one of the calls?

EXERCISES

2.5-1. Define the p.m.f. and give the values of μ, σ^2, and σ when the moment-generating function of X is defined by

(a) $M(t) = 1/3 + (2/3)e^t$.
(b) $M(t) = (0.25 + 0.75e^t)^{12}$.

2.5-2. (i) Give the name of the distribution of X (if it has a name), (ii) find the values of μ and σ^2, and (iii) calculate $P(1 \le X \le 2)$ when the moment-generating function of X is given by

(a) $M(t) = (0.3 + 0.7e^t)^5$.
(b) $M(t) = \dfrac{0.3e^t}{1 - 0.7e^t}, \quad t < -\ln(0.7)$.
(c) $M(t) = 0.45 + 0.55e^t$.
(d) $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t}$.
skip → (e) $M(t) = (0.6e^t)^2(1 - 0.4e^t)^{-2}, \quad t < -\ln(0.4)$.
(f) $M(t) = \sum_{x=1}^{10} (0.1)e^{tx}$.

2.5-3. If the moment-generating function of X is

$$M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

find the mean, variance, and p.m.f. of X.

2.5-4. Let X equal the number of people selected at random that you must ask in order to find someone with the same birthday as yours. Assume that each day of the year is equally likely, and ignore February 29.

(a) What is the p.m.f. of X?
(b) Give the values of the mean, variance, and standard deviation of X.
(c) Find $P(X > 400)$ and $P(X < 300)$.

2.5-5. For each question on a multiple-choice test, there are five possible answers, of which exactly one is correct. If a student selects answers at random, give the probability that the first question answered correctly is question 4.

2.5-6. The probability that a machine produces a defective item is 0.01. Each item is checked as it is produced. Assume that these are independent trials, and compute the probability that at least 100 items must be checked to find one that is defective.

2.5-7. Apples are packaged automatically in 3-pound bags. Suppose that 4% of the time the bag of apples weighs less than 3 pounds. If you select bags randomly and weigh them in order to discover one underweight bag of apples, find the probability that the number of bags that must be selected is

(a) At least 20.
(b) At most 20.
(c) Exactly 20.

2.5-8. Show that 63/512 is the probability that the fifth head is observed on the tenth independent flip of an unbiased coin.

2.5-9. An excellent free-throw shooter attempts several free throws until she misses.

(a) If $p = 0.9$ is her probability of making a free throw, what is the probability of having the first miss on the 13th attempt or later?
(b) If she continues shooting until she misses three, what is the probability that the third miss occurs on the 30th attempt?

2.5-10. Suppose that a basketball player different from the one in Example 2.5-5 can make a free throw 60% of the time. Let X equal the minimum number of free throws that this player must attempt to make a total of 10 shots.

(a) Give the mean, variance, and standard deviation of X.
(b) Find $P(X = 16)$.

2.5-11. Let X equal the number of flips of a fair coin that are required to observe the same face on consecutive flips.

(a) Find the p.m.f. of X. Hint: Draw a tree diagram.
(b) Find the moment-generating function of X.
(c) Use the m.g.f. to find the values of (i) the mean and (ii) the variance of X.
(d) Find the values of (i) $P(X \le 3)$, (ii) $P(X \ge 5)$, and (iii) $P(X = 3)$.
(e) Simulate this experiment to confirm you answers to parts (c) and (d).

2.5-12. Let X have a geometric distribution. Show that

$$P(X > k + j \mid X > k) = P(X > j),$$

where k and j are nonnegative integers. Note: We sometimes say that in this situation there has been loss of memory.

2.5-13. Let X equal the number of flips of a fair coin that are required to observe heads–tails on consecutive flips.

(a) Find the p.m.f. of X. Hint: Draw a tree diagram.
(b) Show that the m.g.f. of X is $M(t) = e^{2t}/(e^t - 2)^2$.

(c) Use the m.g.f. to find the values of (i) the mean and (ii) the variance of X.

(d) Find the values of (i) $P(X \leq 3)$, (ii) $P(X \geq 5)$, and (iii) $P(X = 3)$.

(e) Simulate this experiment to confirm you answers to parts (c) and (d).

2.5-14. Let X equal the number of flips of a fair coin that are required to observe heads on consecutive flips. Let f_n equal the nth Fibonacci number, where $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$, $n = 3, 4, 5, \ldots$.

(a) Show that the p.m.f. of X is

$$f(x) = \frac{f_{x-1}}{2^x}, \qquad x = 2, 3, 4, \ldots.$$

HINT: Draw a tree diagram.

(b) Use the fact that

$$f_x = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^x - \left(\frac{1-\sqrt{5}}{2}\right)^x\right]$$

to show that $\sum_{x=2}^{\infty} f(x) = 1$.

(c) Show that $\mu = E(X) = 6$.

(d) Show that $E[X(X-1)] = 52$, so that the variance of X is $\sigma^2 = 22$.

(e) Find the values of (i) $P(X \leq 3)$, (ii) $P(X \geq 5)$, and (iii) $P(X = 3)$.

(f) Simulate this experiment to confirm your answers to parts (c), (d), and (e). Either write a computer program or physically toss a coin to support the answers.

2.5-15. One of four different prizes was randomly put into each box of a cereal. If a family decided to buy this cereal until it obtained at least one of each of the four different prizes, what is the expected number of boxes of cereal that must be purchased?

2.5-16. Suppose an airport metal detector catches a person with metal 99% of the time. That is, it misses detecting a person with metal 1% of the time. Assume independence of people carrying metal. What is the probability that the first person missed (not detected) is among the first 50 persons scanned?

2.5-17. In 2006, Red Rose tea randomly began placing 1 of 10 English porcelain miniature animals in a 100-bag box of the tea, selecting from 10 "Pet Shop Friends."

(a) On the average, how many boxes of tea must be purchased by a customer to obtain a complete collection consisting of 10 different animals?

(b) If the customer uses one tea bag per day, how long can a customer expect to take, on the average, to obtain a complete collection?

(c) From 2002 to 2006, the figurines were part of the Noah's Ark Animal Series, which contains 15 pieces. Assume again that these figurines were selected randomly. How many boxes of tea would have had to be purchased, on the average, to obtain the complete collection?

2.5-18. If $E(X^r) = 5^r, r = 1, 2, 3, \ldots$, find the moment-generating function $M(t)$ of X and the p.m.f. of X.

2.5-19. Let the moment-generating function $M(t)$ of X exist for $-h < t < h$. Consider the function $R(t) = \ln M(t)$. The first two derivatives of $R(t)$ are, respectively,

$$R'(t) = \frac{M'(t)}{M(t)} \qquad \text{and}$$

$$R''(t) = \frac{M(t)M''(t) - [M'(t)]^2}{[M(t)]^2}.$$

Setting $t = 0$, show that

(a) $\mu = R'(0)$.

(b) $\sigma^2 = R''(0)$.

2.5-20. Use the result of Exercise 2.5-19 to find the mean and variance of the

(a) Bernoulli distribution.

(b) Binomial distribution.

(c) Geometric distribution.

(d) Negative binomial distribution.

2.5-21. Given a random permutation of the integers in the set $\{1, 2, 3, 4, 5\}$, let X equal the number of integers that are in their natural position. The moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120}e^t + \frac{20}{120}e^{2t} + \frac{10}{120}e^{3t} + \frac{1}{120}e^{5t}.$$

(a) Find the mean and variance of X.

(b) Find the probability that at least one integer is in its natural position.

(c) Draw a graph of the probability histogram of the p.m.f. of X.

2.5-22. The probability that a company's workforce has no accidents in a given month is 0.7. The numbers of accidents from month to month are independent. What is the probability that the third month in a year is the first month that at least one accident occurs?

EXERCISES

2.6-1. Let X have a Poisson distribution with a mean of 4. Find

Do exactly by hand and/use R for approximation.

(a) $P(2 \le X \le 5)$.

(b) $P(X \ge 3)$.

(c) $P(X \le 3)$.

2.6-2. Let X have a Poisson distribution with a variance of 3. Find $P(X = 2)$.

2.6-3. Customers arrive at a travel agency at a mean rate of 11 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, give the probability that more than 10 customers arrive in a given hour.

2.6-4. If X has a Poisson distribution such that $3P(X = 1) = P(X = 2)$, find $P(X = 4)$.

2.6-5. Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

2.6-6. A certain type of aluminum screen that is 2 feet wide has, on the average, one flaw in a 100-foot roll. Find the probability that a 50-foot roll has no flaws.

2.6-7. With probability 0.001, a prize of \$499 is won in the Michigan Daily Lottery when a \$1 straight bet is placed. Let Y equal the number of \$499 prizes won by a gambler after placing n straight bets. Note that Y is $b(n, 0.001)$. After placing $n = 2000$ \$1 bets, the gambler is behind if $\{Y \le 4\}$. Use the Poisson distribution to approximate $P(Y \le 4)$ when $n = 2000$.

2.6-8. Suppose that the probability of suffering a side effect from a certain flu vaccine is 0.005. If 1000 persons are inoculated, find the approximate probability that

(a) At most 1 person suffers.

(b) 4, 5, or 6 persons suffer.

2.6-9. A store selling newspapers orders only $n = 4$ of a certain newspaper because the manager does not get many calls for that publication. If the number of requests per day follows a Poisson distribution with mean 3,

(a) What is the expected value of the number sold?

(b) How many should the manager order so that the chance of running out is less than 0.05?

2.6-10. The mean of a Poisson random variable X is $\mu = 9$. Compute

$$P(\mu - 2\sigma < X < \mu + 2\sigma).$$

2.6-11. A Geiger counter was set up in the physics laboratory to record the number of alpha particle emissions of carbon-14 in 0.5 second. Following are 50 observations:

4	6	6	12	11	11	10	5	10	7
9	6	9	11	9	6	4	9	11	8
10	11	7	4	5	8	6	5	8	7
6	3	4	6	4	12	7	14	5	9
7	10	9	6	6	12	10	7	12	13

(a) Calculate the sample mean $\bar{x}$ and sample variance s^2 for these data. Are these two measurements close to each other?

(b) On the same graph, depict the probability histogram for a Poisson distribution with $\lambda = 8$ (we use 8 only because it is the closest value of λ to $\bar{x}$ in Table III in the appendix), with the relative frequency histogram of the data superimposed.

2.6-12. For determining the half-lives of radioactive isotopes, it is important to know what the background radiation is for a given detector over a certain period. A γ-ray detection experiment over 300 one-second intervals yielded the following data:

0	2	4	6	6	1	7	4	6	1	1	2	3	6	4	2	7	4	4	2
2	5	4	4	4	1	2	4	3	2	2	5	0	3	1	1	0	0	5	2
7	1	3	3	3	2	3	1	4	1	3	5	3	5	1	3	3	0	3	2
6	1	1	4	6	3	6	4	4	2	2	4	3	3	6	1	6	2	5	0
6	3	4	3	1	1	4	6	1	5	1	1	4	1	4	1	1	1	3	3
4	3	3	2	5	2	1	3	5	3	2	7	0	4	2	3	3	5	6	1
4	2	6	4	2	0	4	4	7	3	5	2	2	3	1	3	1	3	6	5
4	8	2	2	4	2	2	1	4	7	5	2	1	1	4	1	4	3	6	2
1	1	2	2	2	2	3	5	4	3	2	2	3	3	2	4	4	3	2	2
3	6	1	1	3	3	2	1	4	5	5	1	2	3	3	1	3	7	2	5
4	2	0	6	2	3	2	3	0	4	4	5	2	5	3	0	4	6	2	2
2	2	2	5	2	2	3	4	2	3	7	1	1	7	1	3	6	0	5	3
0	0	3	3	0	2	4	3	1	2	3	3	3	4	3	2	2	7	5	3
5	1	1	2	2	6	1	3	1	4	4	2	3	4	5	1	3	4	3	1
0	3	7	4	0	5	2	5	4	4	2	2	3	2	4	6	5	5	3	4

Do these look like observations of a Poisson random variable with mean $\lambda = 3$? To help answer this question, do the following:

(a) Find the frequencies of $0, 1, 2, \ldots, 8$.

(b) Calculate the sample mean and sample variance. Are they approximately equal to each other?

(c) Construct a probability histogram with $\lambda = 3$ and a relative frequency histogram on the same graph.

(d) What is your conclusion? (Later in this book we consider tests of hypotheses, which help give a statistical answer.)

2.6-13. Let X equal the number of telephone calls per hour that are received by 911 between midnight and noon and that are reported in the *Holland Sentinel*. On October 29 and October 30, the following numbers of calls were reported:

Oct. 29:	0	1	1	1	0	1	2	1	4	1	2	3
Oct. 30:	0	3	0	1	0	1	1	2	3	0	2	2

(a) Calculate the sample mean and sample variance for these data. Are they approximately equal to each other? (See Exercise 2.3-8.)

take λ to be the avg of $\bar{X}$ and s^2

(b) Assume that $\lambda = 1.3$ so that Table III in the appendix can be used. Draw a probability histogram for the Poisson distribution and a relative frequency histogram of the data on the same graph. and use R.

(c) On the basis of these limited data, does it look like the Poisson distribution with $\lambda = 1.3$ could be a reasonable probability model?

2.6-14. Let X equal the number of green peanut M&M's in packages of size 22. Forty-five observations of X yielded the following frequencies for the possible outcomes of X:

Outcome (x):	0	1	2	3	4	5	6	7	8	9
Frequency:	0	2	4	5	7	9	8	5	3	2

(a) Use these data to construct a relative frequency histogram and superimpose over it a Poisson probability histogram with a mean of $\lambda = 5$.

(b) Do these data appear to be observations of a Poisson random variable?

2.6-15. A student trainer worked in the training room in the physical education building. The number of students coming in to be taped or for treatment of other injuries was recorded each half hour, yielding the following data:

3	3	2	0	4	5	6	4	4	3	2	1	2	3	0
5	5	3	2	3	5	4	1	2	0	3	2	4	2	6

(a) Find the sample mean, $\bar{x}$.

(b) Find the sample variance, s^2.

(c) Find the sample standard deviation, s.

(d) Draw a relative frequency histogram of these data with an appropriate Poisson probability histogram superimposed.

2.6-16. A lot (collection) of 10,000 items is accepted if a sample of $n = 400$ has no more than $Ac = 3$ defective items. Determine the approximate values of $OC(0.002)$, $OC(0.004)$, $OC(0.006)$, $OC(0.01)$, and $OC(0.02)$, and plot the operating characteristic curve.

2.6-17. Consider a lot (collection) of 5000 items. Design a sampling plan (i.e., find n and Ac) such that $OC(0.02) = 0.95$ and $OC(0.08) = 0.10$, approximately.

HINT: Start with $n = 50$ and $Ac = 2$ and modify as necessary, noting that increasing n creates a steeper OC curve.

2.6-18. A baseball team loses \$10,000 for each consecutive day it rains. Say X, the number of consecutive days it rains at the beginning of the season, has a Poisson distribution with mean 0.2. What is the expected loss before the opening game?

2.6-19. An airline always overbooks if possible. A particular plane has 95 seats on a flight in which a ticket sells for \$300. The airline sells 100 such tickets for this flight.

(a) If the probability of an individual not showing up is 0.05, assuming independence, what is the probability that the airline can accommodate all the passengers who do show up?

(b) If the airline must return the \$300 price plus a penalty of \$400 to each passenger that cannot get on the flight, what is the expected payout (penalty plus ticket refund) that the airline will pay?

2.6-20. Assume that a policyholder is four times more likely to file exactly two claims as to file exactly three claims. Assume also that the number X of claims of this policyholder is Poisson. Determine the expectation $E(X^2)$.

2.6-21. In the decade of the 1980s, the number X of cases of diphtheria reported each year in the United States followed a Poisson distribution with mean 2.5. What is the probability that

(a) No cases were reported in a given year?

(b) Exactly three cases were reported?

(c) At least three cases were reported?

(d) At most three cases were reported?

2.6-22. For determining half-lives of radioactive isotopes, it is important to know what the background radiation is for a given detector over a certain period. A γ-ray detection experiment over 98 ten-second intervals (observations of a random variable X) yielded the following data:

58	50	57	58	64	63	54	64	59	41	43	56	60	50
46	59	54	60	59	60	67	52	65	63	55	61	68	58
63	36	42	54	58	54	40	60	64	56	61	51	48	50
60	42	62	67	58	49	66	58	57	59	52	54	53	53
57	43	73	65	45	43	57	55	73	62	68	55	51	55
53	68	58	53	51	73	44	50	53	62	58	47	63	59
59	56	60	59	50	52	62	51	66	51	56	53	59	57

(a) Find the sample mean.

(b) Find the sample variance.