Dimensional Analysis Steps

In general the algebraic equation $a = a \times n$ is true only if $n = 1$.

Factors equal to 1 (one) can be made from any equality $a = b$ by dividing both sides of the equation by either a or b. $\frac{a}{a} = \frac{b}{a} = 1$ or $\frac{a}{b} = \frac{b}{b} = 1$.

These things are useful for solving problems involving dimensions by using the following steps. Consider the problem:

$$42.0\text{ in} = ?\text{ ft}$$

Step 1) Write a true equation.

$$42.0\text{ in} = 42.0\text{ in}$$

Step 2) Create a factor of 1 to change units.

12 in = 1 ft so $\frac{12\text{in}}{12\text{in}} = \frac{1\text{ft}}{12\text{in}} = 1$

step 3) Multiply by the equation by the factor.

$$42.0\text{ in} = 42.0\text{ in} \times \left(\frac{1\text{ft}}{12\text{in}}\right)$$

step 4) Cancel the same units in both the numerator and denominator and multiply.

$$42.0\text{ in} = 42.0\text{ in} \times \left(\frac{1\text{ft}}{12\text{in}}\right) = 3.50\text{ ft}$$

Notice that $\frac{12\text{in}}{1\text{ft}}$ is also a factor = 1

so $42.0\text{ in} = 42.0\text{ in} \times \left(\frac{12\text{in}}{1\text{ ft}}\right)$ is true but it is not useful, because the units do not cancel.

So effectively the steps to dimensional analysis are:

(2) $42.0\text{ in} = 42.0\text{ in} \times \left(\frac{\quad}{\text{in}}\right)$ *since the units must cancel.*

(3) $42.0\text{ in} = 42.0\text{ in} \times \left(\frac{1\text{ft}}{12\text{in}}\right)$ *numerator must equal denominator, 1 ft = 12 in*

(4) $42.0\text{ in} = 42.0\text{ in} \times \left(\frac{1\text{ft}}{12\text{in}}\right) = 3.50\text{ ft}$ *answer is correct only if units check out: in×ft/in = ft*

Solving Word Problems by Dimensional Analysis

Problem: 1 ream of paper (500 sheets) is 1.45 inches thick. How many sheets are in a 2.50 inch thick stack of paper?

Step a) Rewrite the question in the form $\underline{\hspace{2cm}} = \ ?\ \underline{\hspace{1cm}}$

The item after the "?" should answer the question "how many?" or "how many what?" In the present example the question says: "How many sheets…?" so:

Step a1) $\underline{\hspace{2cm}} = \text{? sheets}$

Then fill in the starting quantity (left of = sign) **using a quantity in the question**.

Step a2) $2.50 \text{ in} = \text{? sheets}$

Step b) Follow standard methods for dimensional analysis using the other information given as conversion factors.

Notice that "is" (or "are") means "=". In the example problem: $500 \text{ sheets} = 1.45 \text{ in}$

So dimensional analysis steps are:

$$2.50 \text{ in} = \text{? sheets}$$

$$2.50 \text{ in} = 2.50 \text{ in} \times \left(\frac{\underline{\hspace{1.5cm}}}{\text{in}} \right)$$

$$2.50 \text{ in} = 2.50 \text{ in} \times \left(\frac{500 \text{ sheets}}{1.45 \text{ in}} \right)$$

$$2.50 \text{ in} = 2.50 \text{ in} \times \left(\frac{500 \text{ sheets}}{1.45 \text{ in}} \right) = 862. \text{ sheets}$$

Notice also that "per" means equal in forming conversion factors:

4.5 grams per mL or 4.5 g/mL means $4.5 \text{ g} = 1 \text{ mL}$

Notice also that when there is more than one number **in the question**, one must select the starting quantity consistent with the answer desired. If the answer desired is an extensive property (depending upon amount of substance), then the starting quantity must be extensive and the intensive property can be used as a conversion factor. If the answer desired is intensive (independent of amount), then the starting quantity must be intensive. An intensive quantity cannot be converted to an extensive property using dimensional analysis conversions.

Converting Metric Units

Problem 1: $4.0\text{x}10^{21}$ pm = ? Gm

It is possible, but not advisable, to convert directly from pm $\rightarrow$ Gm. It is much safer to do the conversion in two steps: pm $\rightarrow$ m (meters, the base unit) then m $\rightarrow$ Gm.

$$4.0\text{x}10^{21}\text{ pm} = 4.0\text{x}10^{21}\text{ pm} \times \left(\frac{1\text{x}10^{-12}\text{ m}}{1\text{pm}}\right) \times \left(\frac{1\text{Gm}}{1\text{x}10^{9}\text{ m}}\right) = 4.0\text{ Gm}$$

Notice that the conversion factor $\frac{1\text{x}10^{-12}\text{ m}}{1\text{ pm}}$ comes directly from the equality 1 pm = $1\text{x}10^{-12}$ m.

The equality 1 pm = $1\text{x}10^{-12}$ m is obtainable directly from the obvious equality 1 pm = 1 pm by substituting the meaning of p, 10^{-12}, for one p: 1 pm = 10^{-12} m = $1\text{x}10^{-12}$ m.

Likewise 1 Gm = 1 Gm. Then since G means 10^{9}, 1 Gm = 10^{9} m = $1\text{x}10^{9}$ m so $\frac{1\text{Gm}}{1\text{x}10^{9}\text{ m}} = 1$.

Problem 2: $7.0\text{x}10^{25}\text{ nm}^3$ = ? kL

$$7.\text{x}10^{25}\text{ nm}^3 = 7.\text{x}10^{25}\text{ nm}^3 \times \left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right)^3 \times \left(\frac{1\text{cm}}{10^{-2}\text{ m}}\right)^3 \times \frac{1\text{mL}}{1\text{cm}^3} \times \frac{10^{-3}\text{ L}}{1\text{mL}} \times \frac{1\text{kL}}{10^{3}\text{ L}} = 7.\text{x}10^{-2}\text{ kL}$$

Since 1 nm = 1 nm = 10^{-9} m, $\frac{10^{-9}\text{ m}}{1\text{nm}} = 1$. Since $1^3 = 1$, $\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right)^3 = 1$.

Notice when $\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right)$ is cubed, everything inside the parentheses is cubed:

$\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right)^3 = \frac{\left(10^{-9}\right)^3\text{ m}^3}{1^3\text{ nm}^3} = \frac{10^{-27}\text{ m}^3}{1\text{ nm}^3}$. 10^{-9} cubed = $(10^{-9})^3 = 10^{-9}\times10^{-9}\times10^{-9} = 10^{-27}$.

An example that illustrates cubing everything inside (): $6^3 = 216$ or $(2\times3)^3 = 2^3\times3^3 = 8\times27 = 216$

Since $\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right)^3$ gives nm^3 in the denominator and m^3 in the numerator, it converts $\text{nm}^3 \rightarrow \text{m}^3$.

Likewise, $\left(\frac{1\text{cm}}{10^{-2}\text{ m}}\right)^3$ converts $\text{m}^3 \rightarrow \text{cm}^3$; $\left(\frac{1\text{cm}}{10^{-2}\text{ m}}\right)^3 = \frac{1\text{cm}^3}{10^{-6}\text{ m}^3}$. Then by definition

1 cm^3 = 1 mL, so the factor $\frac{1\text{mL}}{1\text{cm}^3} = 1$, does not need to be cubed to convert $\text{cm}^3 \rightarrow$ mL.